Contours and Vector Fields with Mathematica

Below you will find samples of code to help create and better understand contour plots and vector fields. Don't forget that you can press "Ctrl" while dragging to zoom in on RealTime3D graphs.

Getting Started

Type and execute (Shift-Enter) the following to load some packages that we will need.
Remember: No spaces and use backwards apostrophes.

```
<<Calculus`VectorAnalysis`
<<Graphics`PlotField`
<<Graphics`PlotField3D`
<<Graphics`ContourPlot3D`
<<RealTime3D`
SetCoordinates[Cartesian[x,y,z]]
```

If done correctly, the only output will be "`Cartesian[x,y,z]`"

Curves and Vector Fields (2D)

Consider $\int_C (2x - y)\,dx + (x + 3y)\,dy$ where C is the path $x = t$, $y = 2t^2$ from (0, 0) to (2, 8).

What does this look like?

```
curve=ParametricPlot[{t,2t^2},{t,0,2}];
field=PlotVectorField[{2x-y,x+3y},{x,0,2},{y,0,8},
     AspectRatio->1,ScaleFactor->1,Axes->True];
Show[field,curve]
```

Curves and Vector Fields (3D)

Consider $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$ and C is $\mathbf{r}(t) = \sin t\,\mathbf{i} + \cos t\,\mathbf{j} + t^2\mathbf{k}$ for $0 \le t \le \pi/2$.

What does this look like?

```
curve=ParametricPlot3D[{Sin[t],Cos[t],t^2},{t,0,Pi/2}];
field=PlotVectorField3D[{x^2,y^2,z^2},{x,0,1},{y,0,1},{z,0,2.5},
     VectorHeads->True,PlotPoints->10];
Show[curve,field]
```

Zoom in to see it better.

Functions, Contours, and Gradient Fields (3D)

Let's take a look at the function $f(x,y) = x^2y - y^3$.

```
Plot3D[x^2y-y^3,{x,-4,4},{y,-4,4}]
```

To graph the level curves: **`levels=ContourPlot[x^2y-y^3,{x,-4,4},{y,-4,4}]`**

To see them better, add the following options *inside the last* **`]`**. *Don't forget the commas.*

```
     ,ContourShading->False, Contours->35,PlotPoints->40
```

Find the gradient of *f*. Type and execute **`Grad[x^2y-y^3]`**

Graph the gradient: **`field=PlotVectorField[{2x*y,x^2-3y^2},{x,-4,4},{y,-4,4}]`**

Graph both together: **`Show[levels,field]`**

What do you notice?

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Functions, Contours, and Gradient Fields (4D)

Consider the function $f(x,y,z) = x^2 + y^2 + z^2$.
This is now a 4-dimensional situation, we cannot graph it. We can, however, understand the function by graphing the level surfaces. Let's start with the level surface when $f(x,y,z) = 4$.

```
levels=ContourPlot3D[x^2+y^2+z^2,{x,-3,3},{y,-3,3},{z,-3,3},Contours->{4}]
```

To show more level surfaces, change `Contours->{4}` to **`Contours->{1,2,3,4,5}`**

To smooth it, add the option **`,PlotPoints->5`** (any higher greatly increases computation time.)
Where are the level surfaces? Change the domain for *y* to **`{y,-3,0}`**

Find the gradient of *f*. Grad[**`x^2+y^2+z^2`**]
Graph the vector field: **`field=PlotVectorField3D[%,{x,-3,3},{y,-3,0},{z,-3,3}]`**
The "%" means to insert the previous answer here, like "Ans" on a graphing calculator.
Go back and add the option **`,VectorHeads->True`** *inside the last* **`]`** and re-execute.
Oops! It used the graph as %. So, either re-execute the `Grad` command or type it in manually.

All together now: **`Show[levels,field]`**
Be sure to zoom in closely. *What do you notice?*
Try changing the gradient field domain to **`{x,-2,2},{y,-2,0},{z,-2,2}`**

Additional Practice

- Use what you learned above to graph the vector field $\mathbf{F}(x, y, z) = y\mathbf{i} + z\mathbf{j} + x\mathbf{k}$.
- Try creating your own vector fields.
- What happens if one or more of the components is constant?
- Try $\mathbf{F}(x, y, z) = 0\mathbf{i} + x^2\mathbf{j} + 0\mathbf{k}$.
- Repeat the section "Surfaces, Contour Lines, and Gradient Fields (4D)" with the function:

$$f(x,y,z) = \frac{x^2}{3} + \frac{y^2}{4} - z$$

Or try using the following code so you can change the function and plot range easily.

```
f[x_,y_,z_] = x^2/3 + y^2/4 - z;
r = 3;
levels = ContourPlot3D[f[x,y,z],{x,-r,r},{y,-r,r},{z,-r,r},
     Contours ->{0,1,2,3,4,5},PlotPoints ->4];
Grad[f[x,y,z]];
field = PlotVectorField3D[%,{x,-r,r},{y,-r,r},{z,-r,r},
     VectorHeads->True];
Show[levels,field]
```

Try using the code for $f(x,y,z) = x^3 - yz^2$