

# Surface Curves with Mathematica

This lesson and sample code shows you how to draw a curve that is on a given surface.

First, open the following packages:

```
<<Graphics`Shapes`  
<<Graphics`Animation`  
<<RealTime3D`
```

## Curves on a Sphere

Draw a sphere:

```
sphere1=ParametricPlot3D[  
  {Cos[theta]Sin[phi], Sin[theta]Sin[phi], Cos[phi]},  
  {theta, 0, 2Pi}, {phi, 0, Pi}, Boxed->False, Axes->False]
```

In this sphere, “theta” and “phi” are acting as the two parameters for the surface. To make a curve that lies on this surface, convert each surface parameter to a function of  $t$ .

Define:

```
theta[t_]=t;  
phi[t_]=t;  
spherecurve=ParametricPlot3D[  
  {Cos[theta[t]]Sin[phi[t]], Sin[theta[t]]Sin[phi[t]], Cos[phi[t]]},  
  {t, 0, 1}, Boxed->False, Axes->False]
```

Notice that “theta” is now “theta[t]” and “phi” is “phi[t]” and the domain has changed for  $t$ .

Type and execute: `solidcombo=Show[sphere1, spherecurve]`

Change  $theta$  and  $phi$  to `theta[t_]=10Pi*t` and `phi[t_]=Pi*t`; and re-execute.

Add the option `, PlotPoints->200` inside the plot command for `spherecurve` and re-execute.

Try: `SpinShow[spherecurve, Frames->20]` and double-click on one of the images.

You can “stack” the images by double-clicking the middle blue bracket on the right side of the window.

The animation controls are on the bottom left of the window.

Try the following changes:

```
theta(t) = 25πt and phi(t) = 2πt  
theta(t) = cos(5t) and phi(t) = 4t with 0 ≤ t ≤ 2π  
Experiment with your own functions.
```

Go back to the ParametricPlot3D command for `spherecurve` and type 1.1 in front of each of the vector components. This will make the radius of the curve slightly larger than the sphere. Be sure to show both the sphere and your new curve together.

- Continued -

## Curves on a Cone

To draw a cone, let's set this up so it will be easier to change the surface.

```
x[u_,v_]=u*Cos[v];
y[u_,v_]=u*Sin[v];
z[u_,v_]=u;
cone1=ParametricPlot3D[{x[u,v],y[u,v],z[u,v]},{u,-1,1},
{v,0,2Pi},Boxed->False]
```

To create a curve, copy and paste that same code to a new part beneath the cone.

Add the following two lines to the top:

```
u[t_]=t;
v[t_]=10Pi*t; and delete the x, y, and z functions.
```

Change “**cone1**” to “**conecurve**,” make each  $u$  and  $v$  in the ParametricPlot3D command a function of  $t$  by inserting “[**t**]” and change to domain for  $t$  to go from -1 to 1.

Include the option **,PlotPoints->200** in the plot command as well.

Try the following for  $u(t)$  and  $v(t)$ :

$u(t) = \cos(4\pi t)$  and  $v(t) = 3\pi t$  and make the radius of the curve 1.2, show *cone1* and *conecurve* together.

## Curves on Other Surfaces

The parametric equations of a torus are listed below. Use these to create curves on a torus. See if you can “raise” the curve just slightly above the surface.

$$x(u, v) = (2 + \sin v) \cos(u), \quad y(u, v) = (2 + \sin v) \sin(u), \quad z(u, v) = (2 + \cos v)$$

Try:  $x(u, v) = (1/u) \cos(v)$ ,  $y(u, v) = (1/u) \sin(v)$ ,  $z(u, v) = u$  Can you predict what it will look like?

Create curves on paraboloids and cylinders too.

Design your own surfaces and create curves that fit on them.

Copy and paste your favorite images into a new *Mathematica* file for easier demonstration.

## Additional Challenges

Experiment with the different options of **SpinShow**.

Use **<<Default3D`** and **ViewPoint** to create alternate views of your curves.

Experiment with **LightSources** (it's the only way to “color” 3D graphs)